Quantum Entanglement of Electromagnetic Field in Non-inertial Reference Frames

Yi Ling*
Center for Gravity and Relativistic Astrophysics,
Department of Physics, Nanchang University, Nanchang, 330047, China
CCAST (World Laboratory), P.O. Box 8730, Beijing, 100080, China

Song He
Institute of Theoretical Physics, School of Physics, Peking University, Beijing, 100871, China

Weigang Qiu
Department of Physics, Huzhou Teachers College, Huzhou, 313000, China

Hongbao Zhang
Department of Astronomy, Beijing Normal University, Beijing, 100875, China
Department of Physics, Beijing Normal University, Beijing, 100875, China
CCAST (World Laboratory), P.O. Box 8730, Beijing, 100080, China
(Dated: February 1, 2008)

Recently relativistic quantum information has received considerable attention due to its theoretical importance and practical application. Especially, quantum entanglement in non-inertial reference frames has been studied for scalar and Dirac fields. As a further step along this line, we here shall investigate quantum entanglement of electromagnetic field in non-inertial reference frames. In particular, the entanglement of photon helicity entangled state is extensively analyzed. Interestingly, the resultant logarithmic negativity and mutual information remain the same as those for inertial reference frames, which is completely different from that previously obtained for the particle number entangled state.

PACS numbers: 03.67.Mn 03.65.Vf 03.65.Yz

I. INTRODUCTION

Quantum entanglement is both the central concept and the major resource in quantum information science such as quantum teleportation and quantum computation[1]. In recent years, tremendous progress has been made in the research on quantum entanglement: not only have remarkable results been obtained in this field, but also important techniques been applied to various circumstances[2].

Especially, considerable effort has been expended on the investigation of quantum entanglement in the relativistic framework recently[3, 4, 5]. A key issue in this intriguing and active research direction is whether quantum entanglement is observer-dependent. It has been shown that quantum entanglement remains invariant between inertial observers with relative motion in flat spacetime although the entanglement between some degrees of freedom can be transferred to others[6, 7, 8, 9]. However, for scalar and Dirac fields, the degradation of entanglement will occur from the perspective of a uniformly accelerated observer, which essentially originates from the fact that the event horizon appears and Unruh effect results in a loss of information for the non-inertial observer[10, 11, 12, 13].

As a further step along this line, this paper will provide an analysis of quantum entanglement of electromagnetic field in non-inertial reference frames. In particular, we here choose the photon helicity entangled state $\frac{1}{\sqrt{2}}(|\uparrow\rangle_A|\downarrow\rangle_B + |\downarrow\rangle_A|\uparrow\rangle_B)$ rather than the particle number entangled state $\frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$ in an inertial reference frame as our main point for investigation of quantum entanglement in non-inertial reference frames, where A and B represent an inertial observer Alice, and a uniformly accelerated observer Bob respectively, as is illustrated in FIG.1. It thus makes the present work acquire much interest and significance: the former entangled state seems to be more popular in quantum information science, but previous work only restricts within the latter setting[10, 11, 12, 13]. In addition, the result obtained here shows that although Bob is forced to trace over a causally disconnected region of spacetime that he can not access due to his acceleration, which also leads his description of the helicity entangled state to take the form of a mixed state; the corresponding logarithmic negativity and mutual information both remain invariant against the acceleration of Bob. Therefore our result is of remarkable novelty: it is completely different from those obtained for the case of the particle number entangled state, where the degradation of entanglement is dependent on the acceleration of observer, namely, the larger the acceleration, the larger the degradation[10, 11, 12, 13].

The paper is organized as follows. In the next sec-

*Electronic address: yling@ncu.edu.cn

tion, we shall briefly review the four disconnected sectors in Minkowski spacetime and the accelerated observers in Rindler spacetime. In the subsequent section, introducing the two sets of expansion bases for quantizing the electromagnetic field in Minkowski spacetime, we have developed the relationship between the corresponding annihilation and creation operators in Minkowski spacetime. In Section IV, we shall analyze quantum entanglement of electromagnetic field in non-inertial reference frames, especially for the photon helicity entangled state . Conclusions and discussions are presented in the last section.

System of natural units are adopted: $\hbar = c = 1$. In addition, the metric signature takes $(+,-,-,-)$, and the Lorentz gauge condition $\nabla_a A^a = 0$ is imposed onto the electromagnetic potential in flat spacetime, where Maxwell equation reads

$$\nabla_a \nabla^a A_b = 0. \tag{1}$$

Moreover, the well known inner product is reduced to

$$(A, A') = i \int_{\Sigma} [\nabla^a \bar{A}^b) A'_b - \bar{A}_b \nabla^a A'^b] \epsilon_{acde}, \tag{2}$$

which is gauge invariant and independent of the choice of Cauchy surface Σ[14, 15].

II. ACCELERATED OBSERVERS IN MINKOWSKI SPACETIME

Start from Minkowski spacetime

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2. \tag{3}$$

As is shown in FIG. 1, we perform the coordinate transformations for the four disconnected sectors in Minkowski spacetime, respectively, i.e.,

$$\begin{gathered} R \\ t = \rho \sinh \tau, \qquad x = \rho \cosh \tau, \\ \rho = \sqrt{x^2 - t^2}, \qquad \tau = \tanh^{-1}(\frac{t}{x}), \end{gathered} \tag{4}$$

$$\begin{gathered} L \\ t = \rho \sinh \tau, \qquad x = \rho \cosh \tau, \\ \rho = -\sqrt{x^2 - t^2}, \qquad \tau = \tanh^{-1}(\frac{t}{x}), \end{gathered} \tag{5}$$

$$\begin{gathered} F \\ t = \rho \cosh \tau, \qquad x = \rho \sinh \tau, \\ \rho = \sqrt{t^2 - x^2}, \qquad \tau = \tanh^{-1}(\frac{x}{t}), \end{gathered} \tag{6}$$

$$\begin{gathered} P \\ t = \rho \cosh \tau, \qquad x = \rho \sinh \tau \\ \rho = -\sqrt{t^2 - x^2}, \qquad \tau = \tanh^{-1}(\frac{x}{t}). \end{gathered} \tag{7}$$

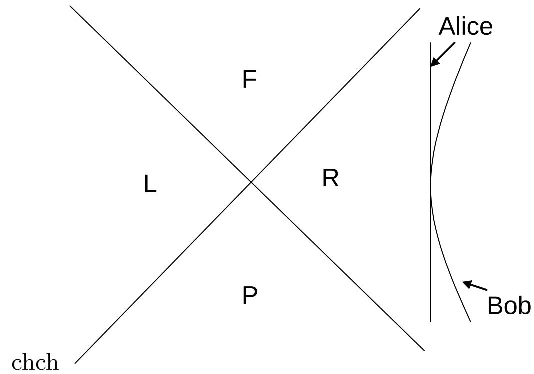

FIG. 1: The four disconnected patches in Minkowski spacetime with an inertial observer Alice and a uniformly accelerated observer Bob constrained in R sector.

In particular, the $R(L)$ sector, viewed as a spacetime in its own right, is also called $R(L)$ Rindler spacetime, where the metric reads

$$ds^2 = \rho^2 d\tau^2 - d\rho^2 - dy^2 - dz^2, \tag{8}$$

and the integral curves of boost Killing field $(\frac{\partial}{\partial \tau})^a$ correspond to the worldlines of accelerated observers with proper time $\rho\tau$ and acceleration $\frac{1}{\rho}$.

III. QUANTUM ELECTROMAGNETIC FIELD IN MINKOWSKI SPACETIME

As is well known, the quantum fields can be expanded in terms of various bases, but the corresponding vacua may be completely different. For the quantum electromagnetic field in Minkowski spacetime, we firstly choose the expansion basis as

$$\begin{aligned} &A_\mu(\omega \in R, p_y \in R, p_z \in R, s = \pm 1) = \\ &\frac{1}{8\pi^2 p_\perp}[(0, 0, p_z\phi, -p_y\phi) + s(\partial_x\phi, \partial_t\phi, 0, 0)], \end{aligned} \tag{9}$$

where $p_\perp = \sqrt{p_y^2 + p_z^2}$, and

$$\phi = \int_{-\infty}^{\infty} d\lambda e^{(-i\omega\lambda - ip_\perp \cosh\lambda t + ip_\perp \sinh\lambda x + ip_y y + ip_z z)} \tag{10}$$

satisfies Klein-Gordon equation in Minkowski spacetime, with ω a dimensionless parameter[14, 16].

It is easy to check that $A_\mu(\omega, p_y, p_z, s)$ is the simultaneous eigensolution of boost, transverse momentum, and helicity operators with the corresponding eigenvalues $\{\omega, p_y, p_z, s\}$ in Minkowski spacetime[15, 17]. Furthermore, it is orthonormal with respect to the inner product (2), i.e.,

$$\begin{aligned} &(A(\omega, p_y, p_z, s), A(\omega', p'_y, p'_z, s')) = \\ &\delta(\omega - \omega')\delta(p_y - p'_y)\delta(p_z - p'_z)\delta_{ss'}. \end{aligned} \tag{11}$$

Thus in terms of this basis, the quantum electromagnetic field can be expanded as

$$\hat{A}_\mu = \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} dp_y \int_{-\infty}^{\infty} dp_z \sum_{s=\pm 1} [c(\omega,p_y,p_z,s)A_\mu(\omega,p_y,p_z,s) + c^\dagger(\omega,p_y,p_z,s)\bar{A}_\mu(\omega,p_y,p_z,s)], \quad (12)$$

where c and $c^\dagger$ are the corresponding annihilation and creation operators, respectively, adjoint to each other, and satisfying the following commutation relations

$$[c(\omega,p_y,p_z,s), c(\omega',p'_y,p'_z,s')] = 0, \quad (13)$$

$$[c^\dagger(\omega,p_y,p_z,s), c^\dagger(\omega',p'_y,p'_z,s')] = 0, \quad (14)$$

$$[c(\omega,p_y,p_z,s), c^\dagger(\omega',p'_y,p'_z,s')] = \delta(\omega-\omega')\delta(p_y-p'_y)\delta(p_z-p'_z)\delta_{ss'}. \quad (15)$$

Next we can also employ Unruh expansion basis for the quantum electromagnetic field, i.e.,

$$R_\mu(\omega\in R^+,p_y,p_z,s) = \frac{1}{\sqrt{2\sinh(\pi\omega)}} [e^{(\frac{\pi\omega}{2})}A_\mu(\omega,p_y,p_z,s) - e^{(-\frac{\pi\omega}{2})}\bar{A}_\mu(-\omega,-p_y,-p_z,s)], \quad (16)$$

$$L_\mu(\omega\in R^+,p_y,p_z,s) = \frac{1}{\sqrt{2\sinh(\pi\omega)}} [e^{(\frac{\pi\omega}{2})}A_\mu(-\omega,p_y,p_z,s) - e^{(-\frac{\pi\omega}{2})}\bar{A}_\mu(\omega,-p_y,-p_z,s)], \quad (17)$$

where R_μ vanishes in the L sector, and L_μ vanishes in the R sector. It is noteworthy that $R_\mu(\omega \in R^+,p_y,p_z,s)(L_\mu(\omega\in R^+,p_y,p_z,s))$ is the simultaneous eigenstate of energy, transverse momentum, and helicity operators with eigenvalues of $\{a\omega,p_y,p_z,s\}$ detected by an observer with uniform acceleration a in the $R(L)$ Rindler spacetime[15, 17]. Moreover, with respect to the inner product (2), Unruh basis is orthonormal, i.e.,

$$(R(\omega,p_y,p_z,s), R(\omega',p'_y,p'_z,s')) = \delta(\omega-\omega')\delta(p_y-p'_y)\delta(p_z-p'_z)\delta_{ss'}, \quad (18)$$

$$(L(\omega,p_y,p_z,s), L(\omega',p'_y,p'_z,s')) = \delta(\omega-\omega')\delta(p_y-p'_y)\delta(p_z-p'_z)\delta_{ss'}, \quad (19)$$

$$(R(\omega,p_y,p_z,s), L(\omega',p'_y,p'_z,s')) = 0. \quad (20)$$

Whence the quantum electromagnetic field can be reformulated as

$$\hat{A}_\mu = \int_{0}^{\infty} d\omega \int_{-\infty}^{\infty} dp_y \int_{-\infty}^{\infty} dp_z \sum_{s=\pm 1} [r(\omega,p_y,p_z,s)R_\mu(\omega,p_y,p_z,s) + r^\dagger(\omega,p_y,p_z,s)\bar{R}_\mu(\omega,p_y,p_z,s) + l(\omega,p_y,p_z,s)L_\mu(\omega,p_y,p_z,s) + l^\dagger(\omega,p_y,p_z,s)\bar{L}_\mu(\omega,p_y,p_z,s)]. \quad (21)$$

Here r and $r^\dagger$ are the corresponding annihilation and creation operators for the R Rindler spacetime; similarly, l and $l^\dagger$ are the corresponding annihilation and creation operators for the L Rindler spacetime. They satisfy the ordinary commutation relations as c and $c^\dagger$ do. Furthermore, they can be related to c and $c^\dagger$ by Bogoliubov transformation, i.e.,

$$r(\omega,p_y,p_z,s) = \frac{1}{\sqrt{2\sinh(\pi\omega)}} [e^{(\frac{\pi\omega}{2})}c(\omega,p_y,p_z,s) + e^{(-\frac{\pi\omega}{2})}c^\dagger(-\omega,-p_y,-p_z,s)], \quad (22)$$

$$l(\omega,p_y,p_z,s) = \frac{1}{\sqrt{2\sinh(\pi\omega)}} [e^{(\frac{\pi\omega}{2})}c(-\omega,p_y,p_z,s) + e^{(-\frac{\pi\omega}{2})}c^\dagger(\omega,-p_y,-p_z,s)]; \quad (23)$$

or vice versa

$$c(\omega,p_y,p_z,s) = \frac{1}{\sqrt{2\sinh(\pi\omega)}} [e^{(\frac{\pi\omega}{2})}r(\omega,p_y,p_z,s) - e^{(-\frac{\pi\omega}{2})}l^\dagger(\omega,-p_y,-p_z,s)], \quad (24)$$

$$c(-\omega,p_y,p_z,s) = \frac{1}{\sqrt{2\sinh(\pi\omega)}} [e^{(\frac{\pi\omega}{2})}l(\omega,p_y,p_z,s) - e^{(-\frac{\pi\omega}{2})}r^\dagger(\omega,-p_y,-p_z,s)]. \quad (25)$$

Note that the vacuum state killed by the annihilation operator c is equivalent to the ordinary Minkowski one[16]. Hence one obtains the expression for the ordinary Minkowski vacuum in the mode $A_\mu(\omega,p_y,p_z,s)$ as a Rindler state, i.e.,

$$|0\rangle^M_{\omega,p_y,p_z,s} = \sqrt{\frac{2\sinh(\pi\omega)}{e^{(\pi\omega)}}} \sum_{n=0}^{\infty} e^{(-n\pi\omega)} |n(\omega,p_y,p_z,s)\rangle^R \otimes |n(\omega,-p_y,-p_z,s)\rangle^L, \quad (26)$$

where $|n(\omega,p_y,p_z,s)\rangle^R(|n(\omega,p_y,p_z,s)\rangle^L)$ denotes the state with n particles in Unruh mode $R_\mu(\omega,p_y,p_z,s)(L_\mu(\omega,p_y,p_z,s))$. Furthermore, we have

$$|1\rangle^M_{\omega,p_y,p_z,s} = c^\dagger(\omega,p_y,p_z,s)|0\rangle^M = [1-e^{(-2\pi\omega)}] \sum_{n=0}^{\infty} e^{-n\pi\omega}\sqrt{n+1} |(n+1)(\omega,p_y,p_z,s)\rangle^R \otimes |n(\omega,-p_y,-p_z,s)\rangle^L \prod_{\{\omega',p'_y,p'_z,s'\}\neq\{\omega,p_y,p_z,s\}} |0\rangle^M_{\omega',p'_y,p'_z,s'}. \quad (27)$$

IV. ENTANGLEMENT FOR ELECTROMAGNETIC FIELDS IN NON-INERTIAL REFERENCE FRAMES

In order to analyze quantum entanglement for electromagnetic field in non-inertial reference frames, firstly following previous work [7, 10, 12, 13], we can also take into account the particle number entangled state in the inertial reference frame associated with Alice, i.e.,

$$|\varphi\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A^M|0\rangle_B^M + |1\rangle_A^M|1\rangle_B^M). \tag{28}$$

It is easy to show that the helicity structure of photon has no influence in this case, and the corresponding calculation goes straightforward, exactly the same as that for scalar particle, which thus justifies modeling photon with scalar particle in investigation of quantum entanglement in non-inertial reference frames for the particle number entangled state[7, 10, 12].

We would next like to concentrate onto two photons' maximally helicity entangled state in the inertial reference frame, i.e.,

$$\begin{aligned}|\psi\rangle &= \frac{1}{\sqrt{2}}(|1\rangle^M_{\omega,p_y,p_z,1A}|1\rangle^M_{\omega,-p_y,-p_z,-1B} \\ &\quad +|1\rangle^M_{\omega,p_y,p_z,-1A}|1\rangle^M_{\omega,-p_y,-p_z,1B}),\end{aligned} \tag{29}$$

which also seems to be more popular than the particle number entangled state in quantum information science. For later convenience, we shall rewrite (29) as

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|1\rangle^M_{+\uparrow A}|1\rangle^M_{-\downarrow B} + |1\rangle^M_{+\downarrow A}|1\rangle^M_{-\uparrow B}). \tag{30}$$

To describe this state from the viewpoint of the non-inertial observer Bob, firstly we shall employ (27) to expand this state. Since Bob is causally disconnected from the L sector, we must take trace over all of the L sector modes, which results in a mixed density matrix between Alice and Bob, i.e.,

$$\begin{aligned}\rho_{AB} = \quad & \frac{[1-e^{(-\frac{2\pi E}{a})}]^2}{2}\sum_{n=0}^{\infty} e^{(-\frac{2n\pi E}{a})}(n+1) \\ & (\ |1\rangle^M_{+\uparrow A}|n+1\rangle^R_{-\downarrow B}\langle 1|^M_{+\uparrow A}\langle n+1|^R_{-\downarrow B} \\ & +|1\rangle^M_{+\uparrow A}|n+1\rangle^R_{-\downarrow B}\langle 1|^M_{+\downarrow A}\langle n+1|^R_{-\uparrow B} \\ & +|1\rangle^M_{+\downarrow A}|n+1\rangle^R_{-\uparrow B}\langle 1|^M_{+\uparrow A}\langle n+1|^R_{-\downarrow B} \\ & +|1\rangle^M_{+\downarrow A}|n+1\rangle^R_{-\uparrow B}\langle 1|^M_{+\downarrow A}\langle n+1|^R_{-\uparrow B} \ ),\end{aligned} \tag{31}$$

where a denotes Bob's acceleration, and $E = a\omega$ is the energy sensitive to Bob's detector.

To determine whether this mixed state is entangled or not, we here use the partial transpose criterion[18]. It states that if the partial transposed density matrix of a system has at least one negative eigenvalue, it must be entangled, otherwise it has no distillable entanglement, but may have other types of entanglement. After a straightforward calculation, the partial transposed density matrix can be obtained as

$$\begin{aligned}\rho^T_{AB} = \quad & \frac{[1-e^{(-\frac{2\pi E}{a})}]^2}{2}\sum_{n=0}^{\infty} e^{(-\frac{2n\pi E}{a})}(n+1) \\ & (\ |1\rangle^M_{+\uparrow A}|n+1\rangle^R_{-\downarrow B}\langle 1|^M_{+\uparrow A}\langle n+1|^R_{-\downarrow B} \\ & +|1\rangle^M_{+\downarrow A}|n+1\rangle^R_{-\downarrow B}\langle 1|^M_{+\uparrow A}\langle n+1|^R_{-\uparrow B} \\ & +|1\rangle^M_{+\uparrow A}|n+1\rangle^R_{-\uparrow B}\langle 1|^M_{+\downarrow A}\langle n+1|^R_{-\downarrow B} \\ & +|1\rangle^M_{+\downarrow A}|n+1\rangle^R_{-\uparrow B}\langle 1|^M_{+\downarrow A}\langle n+1|^R_{-\uparrow B} \ ),\end{aligned} \tag{32}$$

whose eigenvalues are easy to be computed, specifically those belonging to the nth diagonal block are $\frac{[1-e^{[-\frac{2\pi E}{a})]^2}}{2}e^{(-\frac{2n\pi E}{a})}(n+1)(1,1,1,-1)$. Thus the state as seen by Bob will be always entangled if only the acceleration is finite. However, quantification of the distillable entanglement can not be carried out in this case. Therefore we only provide an upper bound of the distillable entanglement by the logarithmic negativity[19]. It is defined as $N(\rho) = \log_2 ||\rho^T||_1$, where$|| \ ||_1$is the trace norm of a matrix. Whence the logarithmic negativity is given by

$$N(\rho_{AB}) = \log_2\{2[1-e^{(-\frac{2\pi E}{a})}]^2\sum_{n=0}^{\infty} e^{(-\frac{2n\pi E}{a})}(n+1)\} = 1, \tag{33}$$

which is independent of the acceleration of Bob.

Further, we can also make an estimation of the total correlation in the state by employing the mutual information, i.e., $I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$ where $S(\rho) = -Tr(\rho\log_2\rho)$ is the entropy of the matrix ρ. According to (31), the entropy of the joint state reads

$$\begin{aligned}S(\rho_{AB}) \quad &= -[1-e^{(-\frac{2\pi E}{a})}]^2\sum_{n=0}^{\infty} e^{(-\frac{2n\pi E}{a})}(n+1) \\ & \log_2\{[1-e^{(-\frac{2\pi E}{a})}]^2 e^{(-\frac{2n\pi E}{a})}(n+1)\}.\end{aligned} \tag{34}$$

Tracing over Alice's states yields Bob's density matrix as

$$\begin{aligned}\rho_B = \quad & \frac{[1-e^{(-\frac{2\pi E}{a})}]^2}{2}\sum_{n=0}^{\infty} e^{(-\frac{2n\pi E}{a})}(n+1) \\ & (\ |n+1\rangle^R_{-\downarrow B}\langle n+1|^R_{-\downarrow B} + |n+1\rangle^R_{-\uparrow B}\langle n+1|^R_{-\uparrow B}),\end{aligned} \tag{35}$$

whose entropy is

$$\begin{aligned}S(\rho_B) = \quad & 1-[1-e^{(-\frac{2\pi E}{a})}]^2\sum_{n=0}^{\infty} e^{(-\frac{2n\pi E}{a})}(n+1) \\ & \log_2\{[1-e^{(-\frac{2\pi E}{a})}]^2 e^{(-\frac{2n\pi E}{a})}(n+1)\}.\end{aligned} \tag{36}$$

Similarly, tracing over Bob's states, we obtain Alice's density matrix as

$$\rho_A = \frac{1}{2}(|1\rangle^M_{+\uparrow A}\langle 1|^M_{+\uparrow A} + |1\rangle^M_{+\downarrow A}\langle 1|^M_{+\downarrow A}), \tag{37}$$

which has an entropy $S(\rho_A) = 1$. As a result, the mutual information is $I(\rho_{AB}) = 2$, which is the same for any uniformly accelerated observer, no matter how much the magnitude of acceleration is.

Therefore, as seen by Bob, the helicity entanglement in non-inertial reference frames shows a remarkably interesting behavior, which is obviously different from the case for the particle number entanglement. In particular, the calculable logarithmic negativity and mutual information both remain constant for the photon helicity entangled state, which is in strong contrast to the particle number entangled state, where they both degrade with the increase of acceleration. All of this seems to imply that the photon helicity entangled state is more robust against the perturbation of acceleration or gravitation than the particle number entangled state, thus can be used as a more effective resource for performing some quantum information processing technology.

V. CONCLUSIONS AND DISCUSSIONS

In this paper we have attempted to provide an analysis of quantum entanglement of electromagnetic field in non-inertial reference frames. In particular, we find that the maximally helicity entangled state is a stable state under acceleration in the sense of its logarithmic negativity and mutual information, which is obviously a novel result, completely different from the case for the particle number entangled state.

As is mentioned in the beginning, the major difference between our work and previous ones concerning quantum entanglement in non-inertial frames is that we have considered the helicity entanglement while previous ones only focus on the entanglement in particle number. The helicity structure is special to photons, which is a completely new trait that can not be presented in the case of scalar particles. It is tempting to say that the entanglement of the discrete degrees of freedom is generally different from the particle number entanglement. Especially, the entangled state seems more immune to the destruction of the acceleration or gravitation in discrete degrees of freedom than particle number. To confirm this conjecture, the spin entanglement of Dirac field in non-inertial reference frames is a necessary and important task worthy of further investigation. Since Dirac particle is constrained by Pauli exclusion principle, it is a qubit-qubit system and the evaluation of the corresponding entanglement is much easier, especially the entanglement of formation can be explicitly calculated[20]. Such a detailed analysis of the spin entanglement in non-inertial reference frames and related problems is expected to be reported elsewhere.

Acknowledgements

We would like to give much gratitude to Chopin Soo for his stimulating suggestion on this work and Bo Hu for his figure plotted here. In addition, we gratefully acknowledge Steven J. van Enk for his insightful and helpful comments. Valuable discussions from Paul Alsing, Robert Mann, and Tracy Tessier are also much appreciated. Y. Ling's work is partly supported by NSFC(Nos.10205002 and 10405027) and SRF for ROCS. S. He's work is supported by NSFC(Nos.10235040 and 10421003). W. Qiu's work is supported by NSFC(No.10547116), the Science Research Fund of Huzhou Teachers College(No.KX21001) and the Science Research Fund of Huzhou City(No.KY21022). H. Zhang's work is supported in part by NSFC(Nos.10373003 and 10533010).

[1] The Physics of Quantum Information, Springer-Verlag, 2000, D. Bouwmeester *et al.*(Eds.).
[2] M. B. Plenio and S. Virmani, quant-ph/0504163.
[3] S. J. van Enk and T. Rudolph, Quant. Inf. Comput. 3: 423(2003).
[4] A. Peres and D. R. Terno, Rev. Mod. Phys. 76: 93(2004).
[5] Y. Shi, Phys. Rev. D70: 105001(2004).
[6] A. Peres *et al.*, Phys. Rev. Lett. 88: 230402(2002).
[7] P. M. Alsing and G. J. Milburn, Quant. Inf. Comput. 2: 487(2002).
[8] R. M. Gingrich and C. Adami, Phys. Rev. Lett. 89: 270402(2002).
[9] S. He *et al.*, quant-ph/0701233.
[10] P. M. Alsing and G. J. Milburn, Phys. Rev. Lett. 91: 180404(2003).
[11] P. M. Alsing *et al.*, J. Optics. B6: S834 (2004).
[12] I. Fuentes-Schuller and R. B. Mann, Phys. Rev. Lett. 95: 120404(2005).
[13] P. M. Alsing *et al.*, Phys. Rev. A74: 032326(2006).
[14] V. Moretti, J. Math. Phys. 38: 2922(1997).
[15] Y. Hu *et al.*, J. Math. Phys. 47: 052304(2006).
[16] D. Colosi, Nuovo Cim. B115: 1101(2000).
[17] A. Ashtekar, J. Math. Phys. 27: 824(1986).
[18] A. Peres, Phys. Rev. Lett. 77: 1413(1996).
[19] G. Vidal and R. F. Werner, Phys. Rev. A65: 032314(2002).
[20] W. K. Wootters, Phys. Rev. Lett. 80: 2245(1998).